Lesson 7.2 • Properties of Isometries

In this activity you will investigate isometries on a coordinate plane. Then you’ll demonstrate an important property of reflections.

Investigation 1: Transformations on a Coordinate Plane

Sketch
Step 1 In a new sketch, construct vertices of a letter by holding down the Shift key while using the Point tool. Choose Construct | Interior.
Step 2 Choose Graph | Show Grid. Drag the points of your letter so they fall on grid points.

Investigate
1. You can indicate transformations on the coordinate plane using an ordered pair rule. For example, the transformation shown follows the rule \((x, y) \rightarrow (x + 3, y + 1)\). Notice that the translation vector from point \(K\) to point \(L\) goes to the right three units and up one unit. Use the Point tool to construct the points to which the vertices of your letter will move by the rule \((x, y) \rightarrow (x + 3, y + 1)\). Select in pairs each corner of the letter and the point to which it will move. Then choose Edit | Action Buttons | Movement. Choose to have points move toward their initial destinations. When the action button appears, press it to make your letter move.
2. Change your sketch to demonstrate the transformation \((x, y) \rightarrow (-x, y)\). Explain how to create this transformation and state whether it’s a translation, a reflection, or a rotation.

Investigation 2: Finding a Minimal Path

When you hit a pool ball without putting any spin on it, the ball follows a straight path toward the cushion. It bounces off the cushion at the same angle at which it came in. Can you see how this path is like a reflection?

Sketch
Step 1 In a new sketch, construct a long horizontal segment \(AB\) to represent the cushion.
Step 2 Construct points \(C\) and \(D\) on the same side of the segment. Point \(C\) represents the cue ball, and point \(D\) represents the ball you plan to hit.
Step 3 Construct \(CE\) and \(ED\), where point \(E\) is a point on \(AB\). These segments together show the path of the cue ball with one bounce off the cushion.
Step 4 Measure \(CE\) and \(ED\).
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Step 5  Use the calculator to find the sum of these two lengths.

Step 6  Measure the incoming and outgoing angles the cue ball makes with the cushion (\( \angle CEA \) and \( \angle DEB \)).

Step 7  Double-click \( AB \) to mark it as a mirror. Then reflect point \( D \) over \( AB \) to create point \( D' \).

Investigate

1. Drag point \( E \) and observe the angle measurements. Locate \( E \) at the point where the cue ball should hit the cushion. Explain how you found this point.

2. Construct \( CD' \) and change its line width to dashed using the Display menu. Find its length. Explain how you can use the reflection of point \( D \) to figure out how to aim the cue ball.

3. Now drag point \( E \) again, this time watching the sum \( CE + ED \). How is this total distance related to the path of the cue ball?

4. How can you use reflections to find the minimal path between two points if your path must touch a neighboring line? Write your observations as the Minimal Path Conjecture. Explain why your method works.